rate; Re, Reynolds number; $Re = \rho_\infty U_\infty R/\mu$; C_x, drag coefficient of the disks; $C_x = 4\left(\int_0^r \Delta p y dy + \int_0^R \Delta p y dy\right) / (\rho_\infty U_\infty^2 R^2$; C_p, pressure coefficient, $C_p = 2p/(\rho_\infty U^2_\infty)$.

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EFFECT OF VARIATION OF THE PHYSICAL PROPERTIES OF THE GAS ON THE STABILITY OF LAMINAR NONISOTHERMAL FLOW IN A CHANNEL WITH PERMEABLE WALLS

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UDC 536.24

An investigation is made into the stability relative to small perturbations of a quasideveloped gas flow with variable physical properties in a plane channel in the presence of heating or cooling.

In [1] the flow of a gas with variable physical properties was investigated in the entry region of a plane channel with permeable walls. The flow stability in a channel with permeable walls has been investigated only for isothermal flow of a constant-property fluid, e.g., [2, 3]. We have now analyzed the effect of varying the physical properties of the gas on the quasideveloped flow and its stability in the presence of heating and cooling.

For two-dimensional plane flow the system of equations describing the mass, momentum, and heat transfer has the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_x}{\partial x} + \frac{\partial \rho u_y}{\partial y} = 0, \tag{1}$$

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left\{\mu\left[2\frac{\partial u_x}{\partial x} - \frac{2}{3}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right)\right]\right\} + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right], \tag{2}$$

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$$\rho\left(\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_y}{\partial x}+\right.\right.$$

$$\left.\left.+\frac{\partial u_x}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial u_y}{\partial y}-\frac{2}{3}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)\right]\right\}, \tag{3}$$

$$\rho\left(\frac{\partial h}{\partial t}+u_x\frac{\partial h}{\partial x}+u_y\frac{\partial h}{\partial y}\right)=-\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right). \tag{4}$$

In order to construct a stationary solution of Eqs. (1)-(4) in the region of quasideveloped flow (remote from the entry section) we will employ the local similarity method used in [4] to calculate the flow in a circular tube. Thus, we take

$$u_y=V_w\bar{u}_y(\bar{y}),\quad \frac{\partial u_x}{\partial x}=\frac{\rho_m}{\rho_w}\frac{dU_m}{dx}\bar{u}_x(\bar{y}),\quad \frac{\partial h}{\partial x}=\frac{q_w}{\rho\bar{U}\delta}; \tag{5}$$

here, for a perfect gas satisfying the Clapeyron–Mendeleev equation of state, the variation of the mean velocity $U_m = \rho U/\rho_m$ along the length is equal to

$$\frac{dU_m}{dx}=-\frac{\rho_w V_w}{\rho_m\delta}+\frac{q_w}{C_{pm}T_m\rho_m\delta}. \tag{6}$$

The first term in (6) describes the variation of the mean flow velocity as a result of injection, and the second that resulting from the density changes associated with the heating or cooling of the gas (thermal acceleration or retardation).

Using relations (5) and (6), we reduce system of equations (1)-(4) to ordinary differential equations for the relative velocities $\bar{u}_y$, $\bar{u}_x = u_x\rho_w/\rho U$ and temperature $\bar{T} = T/T_w$:

$$\frac{d}{d\bar{y}}(\bar{\rho}\bar{u}_y)-\bar{\rho}\bar{u}_x=0, \tag{7}$$

$$\frac{d}{d\bar{y}}\left(\bar{\mu}\frac{d\bar{u}_x}{d\bar{y}}\right)-R\bar{\rho}\bar{u}_y\frac{d\bar{u}_x}{d\bar{y}}+(R-K_c\theta_w Q)\bar{\rho}\bar{u}_x^2+K=0, \tag{8}$$

$$\frac{d}{d\bar{y}}\left(\bar{\lambda}\frac{d\bar{T}}{d\bar{y}}\right)-R\,\mathrm{Pr}_w\bar{C}_p\bar{\rho}\bar{u}_y\frac{d\bar{T}}{d\bar{y}}-\mathrm{Pr}_w Q\bar{\rho}\bar{u}_x=0. \tag{9}$$

The boundary conditions for system of equations (7)-(9) take the form

$$\bar{y}=0\ \ \bar{u}_y=\frac{d\bar{u}_x}{d\bar{y}}=\frac{d\bar{T}}{d\bar{y}}=0,\quad \bar{y}=1\ \ \bar{u}_x=0,\quad \bar{u}_y=\bar{T}=1.$$

The calculations were carried out for air ($\mathrm{Pr}_w = 0.7$), the temperature dependences of the thermophysical properties being described by power laws:

$$\bar{\mu}=\bar{T}^{n_\mu},\ \bar{\lambda}=\bar{T}^{n_\lambda},\ \bar{C}_p=\bar{T}^{n_c}\ \text{with}\ n_\mu=0.7;\ n_\lambda=0.8;\ n_c=0.1.$$

In Fig. 1a, b we have plotted the velocity and temperature distributions over the cross section of the channel. Clearly, in both the absence and the presence of injection, heating results in a fuller, and cooling in a less full velocity profile. This deformation of the velocity profile is attributable to the effect of thermal acceleration or retardation of the flow. As with a circular tube [4], in the presence of injection with increase in heating rate there is a shifting of the velocity maximum away from the channel axis toward the wall, which is associated with the acceleration of the light (heated) gas in the boundary region under the influence of the negative pressure gradient induced by injection. The effect of varying the physical properties on the temperature distribution in the channel is qualitatively the same for permeable and impermeable walls and consists in the profile becoming less full on heating and fuller on cooling, i.e., the opposite of the effect on the velocity distribution.

Let us analyze the stability of the quasideveloped flow described by Eqs. (7)-(9) with respect to small two-dimensional perturbations in the hydrodynamic approximation, i.e., without taking into account the fluctuations of the physical properties due to temperature fluctuations. From Eqs. (1)-(3) we obtain the equations for the velocity and pressure perturbations and introduce the stream function for the fluctuation motion

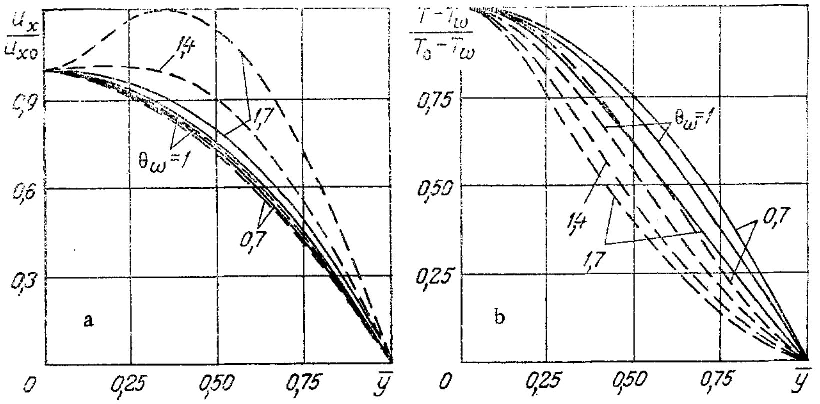

Fig. 1. Velocity (a) and temperature (b) profiles: continuous curves) R = 0; broken curves) R = −5.

$$\overline{\rho u_x'} = \frac{\partial\psi}{\partial\bar y},\quad \overline{\rho u_y'} = -\frac{\partial\psi}{\partial x},$$

$$\psi = \Phi(\bar y)\exp[i\alpha(\bar x - c\bar t)].$$

Here, as the length scale we have used δ, and as the velocity scale $\overline{\rho U}/\rho_w$. The equation for the amplitude of the velocity perturbations Φ is written in the form

$$\frac{d^2}{d\bar y^2}\left\{\bar\mu\left[\frac{d}{d\bar y}\left(\frac{1}{\bar\rho}\frac{d\Phi}{d\bar y}\right)+\frac{\alpha^2}{\bar\rho}\Phi\right]\right\} - 2\alpha^2\frac{d}{dx}\left\{\bar\mu\left[\frac{1}{\bar\rho}\frac{d\Phi}{d\bar y} + \frac{d}{d\bar y}\left(\frac{\Phi}{\bar\rho}\right)\right]\right\} + \alpha^2\bar\mu\left[\frac{d}{d\bar y}\left(\frac{1}{\bar\rho}\frac{d\Phi}{d\bar y}\right)+\frac{\alpha^2}{\bar\rho}\Phi\right] =$$

$$= \frac{i\alpha \mathrm{Re}_m}{K_\mu}\left[(\bar u_x - c)\left(\frac{d^2\Phi}{d\bar y^2}-\alpha^2\Phi\right) - \frac{d^2\bar u_x}{d\bar y^2}\Phi\right] + R\bar u_y\left(\frac{d^3\Phi}{d\bar y^3}-\alpha^2\frac{d\Phi}{d\bar y}\right) + 2\left[(K_c\theta_w Q - R)\bar u_x + R\frac{d\bar u_y}{d\bar y}\right]\left(\frac{d^2\Phi}{d\bar y^2}-\alpha^2\Phi\right) + \left[2(K_c\theta_w Q - R)\frac{d\bar u_x}{d\bar y} + R\frac{d^2\bar u_y}{d\bar y^2}\right]\frac{d\Phi}{d\bar y} = 0. \quad (10)$$

As compared with the usual Orr–Sommerfeld equation, Eq. (10) includes terms containing derivatives of the viscosity and density, and there are also additional terms describing the convective transport of the fluctuations in the transverse direction, as a result of injection, and in the longitudinal direction, as a result of thermal acceleration (retardation) in the course of heating (cooling) and acceleration due to injection. Equation (10) was solved by a differential pivot method [5] with boundary conditions for the symmetrical perturbations

$$\Phi'(0) = \Phi'''(0) = \Phi(1) = \Phi'(1) = 0, \quad (6)$$

which, as a rule, are the most dangerous from the instability standpoint.

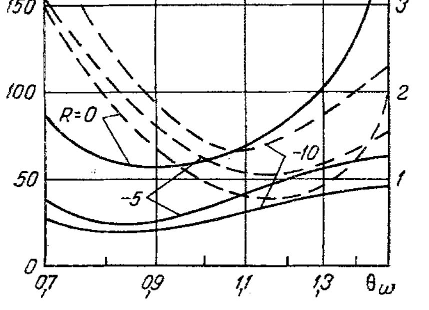

Fig. 2. Critical stability parameters: continuous curves) Re_{cr}; broken curves) α_{cr}.

The stability calculations were carried out on the temperature factor interval $0.7 \leq \theta_w \leq 1.5$ and the injection parameter interval $-10 \leq R \leq 0$. In Fig. 2 we have plotted the results of calculating the critical Reynolds number and critical wave number, obtained from the neutral stability curves, as a function of the temperature factor. Clearly, in both the absence and presence of injection, in the case of heating the critical Reynolds number increases, which is primarily associated with the greater fullness of the velocity profile of the undistrubed laminar flow owing to thermal acceleration. The increase in flow stability on heating is consistent with the experimental data for a circular tube with impermeable walls [6-8], where it was established that when the gas is heated the laminar regime is preserved at Reynolds numbers substantially exceeding the critical value for isothermal flow. As the cooling rate increases we first observe a fall in $Re_{m\ cr}$ due to thermal retardation of the flow after which the destablizing effect of cooling is replaced by a stabilizing effect, probably associated, as in the boundary layer [9], with the variation of the viscosity over the channel cross section. The critical wave number increases with cooling and intense heating, i.e., the flow becomes unstable with respect to shorter-wave perturbations. On the whole, the effect of the change in physical properties during heating and cooling on the stability of flows in channels with impermeable walls is qualitatively the same.

NOTATION

u_x, u_y, longitudinal and transverse velocity components; p, pressure; h, enthalpy; T, temperature; ρ, density; μ, dynamic viscosity; λ, thermal conductivity; C_p, heat capacity; δ, half-width of the channel; V_w, injection velocity ($V_w < 0$); q_w, heat flux density; $\rho U = \int_0^\delta \rho u_x dy/\delta$, mass velocity; $\theta_w = T_w/T_m$, temperature factor; $R = \rho_w V_w \delta/\mu_w$, injection Reynolds number; $Re_m = \overline{\rho U}\delta/\mu_m$, mainstream Reynolds number; α, wave number; c, propagation velocity of the perturbations; $\bar{y} = y/\delta$; $\bar{\rho} = \rho/\rho_w$; $\bar{\mu} = \mu/\mu_w$; $\bar{\lambda} = \lambda/\lambda_w$; $\bar{C}_p = C_p/C_{pw}$; $K_c = C_{pw}/C_{pm}$; $K_\mu = \mu_w/\mu_m$; $Q = q_w\delta/C_{pw}\mu_w T_w$; $K = \rho^2{}_w\delta^2/(\overline{\rho U}\rho_m\mu_w)\partial p/\partial x$. The subscripts w and m relate to parameters determined at the wall temperature T_w and the mass-average temperature T_m.

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